Patterns in the CERES Global Mean Data, Part 4.

From Schwarzschild (1906) to CERES (2019):
Laws Behind The Rules (Patterns Explained)

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NASA LaRC

"Equation (2.17) is known as the equation of transfer, and was first given in this form by Schwarzschild. While it sets the pattern of the formalism used in transfer problems, its physical content is very slight." — Goody and Yung (1989)

"The Eddington approximation will generally be employed; while it is not precise it omits no essential physical principles, provided that the medium is stratified." — Goody (1964)

Ueber das Gleichgewicht der Sonnenatmosphäre

Von **K. Schwarzschild.**

Vorgelegt in der Sitzung vom 13. Januar 1906.

Consider now, at some point in the solar atmosphere, the radiative energy A which is transmitted outward, and the radiative energy B, which (due to the radiation of outer layers) is transmitted inward.

Treat first the inward energy B. When traveling inward through an infinitesimally thin layer dh, the fraction $aBdh$ of B will be lost; on the other hand, the contribution $aEdh$ due to the lateral radiation of the layer itself will be added to B. All in all,

$$\frac{dB}{dh} = a(E - B). \tag{7}$$

In the case of the outward energy A, we procede analogously and obtain

$$\frac{dA}{dh} = -a(E - A). \tag{8}$$

Given the absorption coefficient a as a function of depth h, define the "average optical depth"* of the atmosphere lying above the depth h by

$$\bar{\tau} = \int^h adh. \tag{9}$$

The differential equations then become

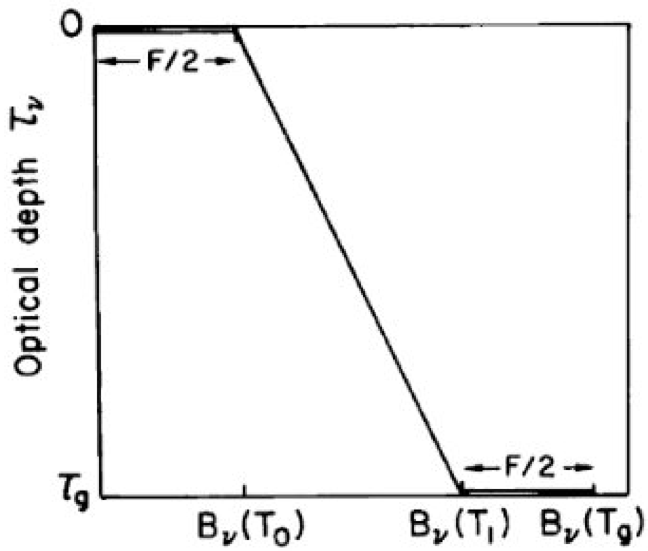
$$\frac{dB}{d\bar{\tau}} = E - B, \qquad \frac{dA}{d\bar{\tau}} = A - E. \tag{10}$$

This leads to the final result

$$E = \frac{A_0}{2}(1 + \bar{\tau}), \qquad A = \frac{A_0}{2}(2 + \bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \tag{11}$$

$$E = \frac{A_0}{2}(1+\bar{\tau}), \qquad A = \frac{A_0}{2}(2+\bar{\tau}), \qquad B = \frac{A_0}{2}\bar{\tau}. \qquad (11)$$

$A - E = \Delta A = A_0/2$ independent of τ



Chamberlain (1978)

Theory of Planetary Atmospheres,
Academic Press

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_\nu(T)$ vs. τ_ν. Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

$$B_g - B_0 = \frac{\phi}{2\pi}$$

Houghton (2002, Eq. 2.13)

The Physics of Atmospheres,
Cambridge Univ Press

$$\boxed{\Delta B_g = B_g - B_0 = B_{eff}/2}$$

9

(Goody and Yung 1989)

ATMOSPHERES IN RADIATIVE EQUILIBRIUM

9.1. Introduction

In this chapter we discuss *radiative equilibrium models* of the earth's atmosphere and the closely related *radiative–convective models*, for which small-scale convection is included in a highly parameterized form. In both cases, heat transports by planetary-scale motions are neglected.

$$B(\tau) = \frac{\sigma\theta(\tau)^4}{\pi} = \frac{-F_S(1+3\tau/2)}{2\pi}$$
$$B^*(\tau_1) = \frac{\sigma\theta_g^4}{\pi} = \frac{-F_S(2+3\tau_1/2)}{2\pi} \qquad (9.5)$$

There are discontinuities,

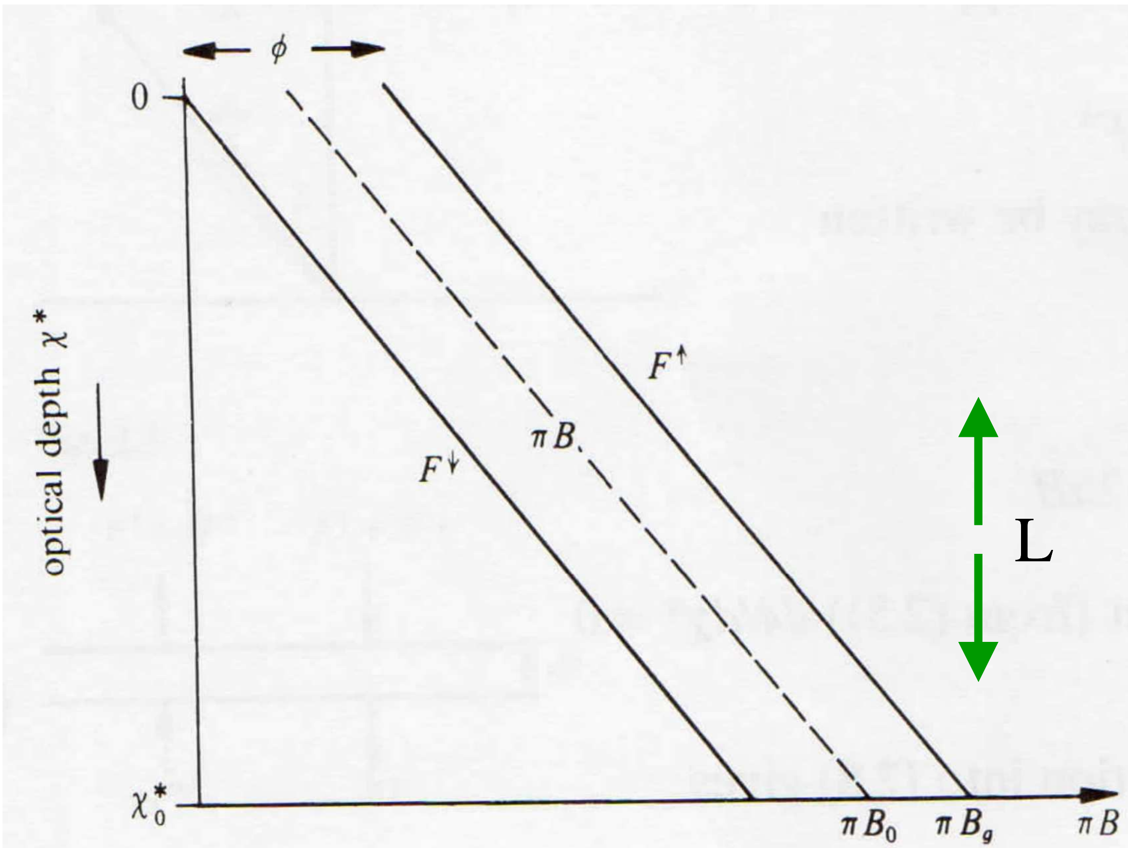
$$\Delta B = \frac{F_S}{2\pi}$$

My Eq. (1) $\Delta B = B_{eff}/2$

The solution, (9.5), although based upon many simplifications, has features that are instructive for planetary atmospheres.

Houghton (2002, Fig. 2.4)

The Physics of Atmospheres, Cambridge Univ Press



Eq. (1) (clear-sky)
$\Delta B = B_{eff}/2$

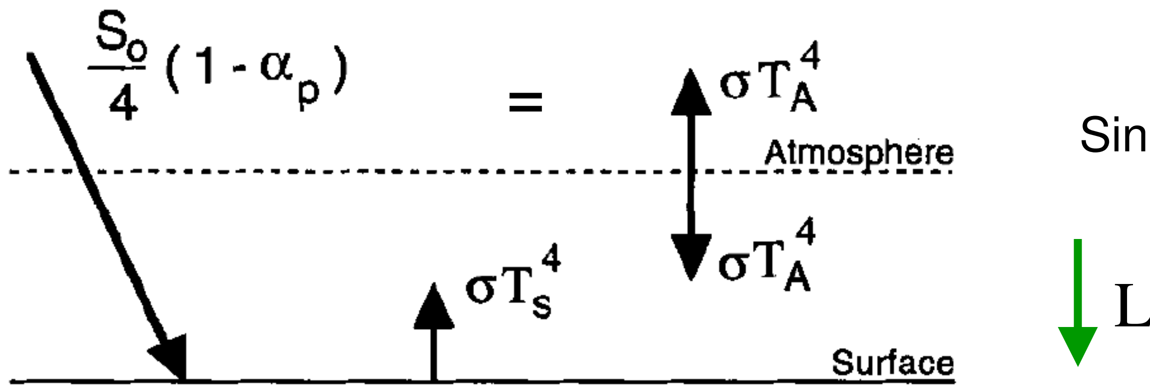
My Eq. (2) (all-sky)
$\Delta B = (B_{eff} - L)2$

Separating atmospheric radiation from longwave cloud effect (L):

Eq. (2): $\Delta B_g = (B_{eff} - L)/2$ (surface net, all-sky)

Hartmann (1994)

Global Physical Climatology



Single-slab

Fig. 2.3 Diagram of the energy fluxes for a planet with an atmosphere that is transparent for solar radiation but opaque to terrestrial radiation.

atmosphere and the surface. The atmospheric energy balance gives

$$\sigma T_s^4 = 2\sigma T_A^4 \quad \Rightarrow \quad \sigma T_s^4 = 2\sigma T_e^4 \tag{2.12}$$

and the surface energy balance is consistent:

$$\frac{S_0}{4}\left(1-\alpha_p\right)+\sigma T_A^4 = \sigma T_s^4 \quad \Rightarrow \quad \sigma T_s^4 = 2\sigma T_e^4 \tag{2.13}$$

Surface total (gross) SW + LW energy income:	**$B_g = 2B_{eff}$**
Adding cloud effect, the surface absorption is:	**$B_g = 2B_{eff} + L$**

Houghton (2002)

$$B = \frac{\phi}{2\pi}(\chi^* + 1) \quad (2.12)$$

At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^{\uparrow} = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0, and

$$B_g - B_0 = \frac{\phi}{2\pi} \quad (2.13)$$

2.5 The greenhouse effect

Combining (2.12) and (2.13) we find that for the radiative equilibrium atmosphere:

$$B_g = \frac{\phi}{2\pi}(\chi_0^* + 2) \quad (2.15)$$

With optical depth $\chi^*_0 = 2$,

My Eq. (3) Surface total, clear-sky: $\pi B_g = 2\phi$

My Eq. (4) With cloud effect, all-sky: $\pi B_g = 2\phi + L$

My four equations

Eq. (1) Schwarzschild (1906, Eq. 11), net, clear-sky

$$A - E = \Delta A = A_0 / 2$$

Eq. (2) Schwarzschild (1906, Eq. 11), incl LWCRE, net, all-sky

$$A - E = \Delta A = (A_0 - L) / 2$$

Eq. (3) Schwarzschild (1906, Eq. 11), at $\tau = 2$, gross, clear-sky

$$A = 2A_0$$

Eq. (4) Schwarzschild (1906, Eq. 11), at $\tau = 2$, incl LWCRE, gross, all-sky

$$A = 2A_0 + L$$

My four equations

Eq. (1): Houghton Eq. (2.13)

Eq. (2): Houghton Eq. (2.13) incl LWCRE

Eq. (3): Houghton Eq. (2.15) at $\chi^*_0 = 2$

Eq. (4): Houghton Eq. (2.15) at $\chi^*_0 = 2$, incl LWCRE

Eq. (1) Surface net, clear-sky: $\Delta B_g = B_g - B_0 = B_{eff}/2$

Eq. (2) Surface net, all-sky: $\Delta B_g = B_g - B_0 = (B_{eff} - L)/2$

Eq. (3) Surface gross, clear-sky: $B_g = 2B_{eff}$

Eq. (4) Surface gross, all-sky: $B_g = 2B_{eff} + L$

The four equations in CERES notation

Eq. (1) ΔB_g = SFC SW net + LW net, clear-sky = OLR/2

Eq. (2) ΔB_g = SFC SW net + LW net, all-sky = (OLR – LWCRE)/2

Eq. (3) B_g = SFC SW net + LW down, clear-sky = 2OLR

Eq. (4) B_g = SFC SW net + LW down, all-sky = 2OLR + LWCRE

Surface LW up (ULW) = LW down + LW net (both for clear and all)

LWCRE at the TOA = LWCRE at the surface

Accuracy of the equations in CERES EBAF Ed4.1, annual global means for 19 running years, 12/2000 – 11/2019

2001	340.166	239.788	266.178	344.674	316.613	397.756	397.695	186.831	23.612	163.218	211.672
2002	340.177	240.337	266.436	345.294	317.346	398.645	398.410	186.273	23.315	162.958	211.883
2003	340.068	240.401	266.273	345.169	317.282	398.513	398.125	186.742	23.375	163.368	211.359
2004	340.013	240.138	265.963	345.212	316.898	398.103	397.862	186.952	23.478	163.474	211.991
2005	339.966	240.251	266.187	346.071	317.737	398.873	398.543	186.302	23.229	163.072	211.479
2006	339.943	240.033	266.044	345.053	317.309	398.400	398.218	186.719	23.154	163.565	211.846
2007	339.914	240.468	266.152	344.670	317.161	398.448	398.228	186.355	23.095	163.260	211.585
2008	339.908	239.765	265.631	343.500	316.110	397.466	397.345	186.920	23.400	163.521	211.567
2009	339.912	239.915	265.778	344.295	317.067	398.124	398.023	186.886	23.372	163.515	211.476
2010	339.968	240.345	266.129	345.444	318.108	398.578	398.428	185.628	23.013	162.614	211.400
2011	340.027	240.038	265.628	343.808	316.484	397.723	397.646	186.399	23.061	163.338	211.577
2012	340.091	239.880	265.623	344.411	316.866	398.165	398.066	186.643	23.105	163.538	211.546
2013	340.083	240.075	265.804	345.163	317.223	398.360	398.238	186.700	23.398	163.302	211.773
2014	340.052	240.248	265.853	345.442	317.603	398.717	398.567	186.961	23.363	163.598	211.676
2015	340.138	240.424	265.925	346.364	318.692	399.428	399.287	186.902	23.114	163.787	211.405
2016	340.038	240.708	266.364	347.201	319.471	400.291	399.944	186.899	22.684	164.217	212.232
2017	339.953	240.610	266.193	346.265	318.313	399.740	399.363	187.320	22.838	164.483	212.387
2018	339.944	240.170	265.812	344.956	317.767	399.339	398.996	187.227	22.975	164.252	212.049
2019	339.942	240.576	266.166	344.940	318.131	400.007	399.733	187.623	22.840	164.783	211.965
	340.02	**240.22**	**266.01**	**345.15**	**317.48**	**398.67**	**398.46**	**186.75**	**23.18**	**163.57**	**211.73**
	isr	olr_a	olr_c	dlr_a	dlr_c	ulw_a	ulw_c	sw_d_a	sw_u_a	swnet_a	swnet_c
		ΔEq1		ΔEq2		ΔEq3		ΔEq4			
		-2.25		**2.84**		**-2.80**		**2.50**			

Accuracy of the equations, EBAF Ed4.1, 19 years

Eq. (1) Clear-sky, net

SFC SW net clear-sky	= 211.73
SFC LW down clear-sky	= 317.48
SFC LW up clear-sky	= 398.46
SFC SW+LW net, clear-sky	**= 130.75**
TOA LW /2, clear-sky	**= 133.00**

ΔEq(1) = – 2.25 Wm^{-2}

Eq. (2) All-sky, net

SFC SW net all-sky	= 163.57
SFC LW down all-sky	= 345.15
SFC LW up all-sky	= 398.67
TOA LW, all-sky	= 240.22
LWCRE	= 25.79
SFC SW+LW net, all-sky	**= 110.05**
(TOA LW – LWCRE)/2	**= 107.21**

ΔEq(2) = 2.84 Wm^{-2}

Eq. (3) Clear-sky, gross

SFC SW net clear-sky	= 211.73
SFC LW down clear-sky	= 317.48
SFC SW net + LW down	**= 529.21**
2TOA LW, clear-sky	**= 532.02**

ΔEq(3) = – 2.80 Wm^{-2}

Eq. (4) All-sky, gross

SFC SW net all-sky	= 163.57
SFC LW down all-sky	= 345.15
TOA LW, all-sky	= 240.22
LWCRE	= 25.79
SFC SW net +LW down, all	**= 508.72**
2TOA LW + LWCRE	**= 506.22**

ΔEq(4) = 2.50 Wm^{-2}

Definitions and integer solution

SFC LW down clear-sky	= SFC LW down all – LWCRE
TOA LW clear-sky	= TOA LW all + LWCRE
LWCRE TOA	= LWCRE SFC
SFC LW up all-sky	= SFC LW up clear-sky

Surface LW up, all-sky	=	**15**	Surface LW up, clear-sky	=	**15**
Surface SW net, all-sky	=	**6**	Surface SW net, clear-sky	=	**8**
Surface LW net, all-sky	=	**–2**	Surface LW net, clear-sky	=	**–3**
Surface SW+LW net, all-sky	=	**4**	Surface SW+LW net, clr-sky	=	**5**
Surface SW+LW gross, all	=	**19**	Surface SW+LW gross, clear	=	**20**
Surface LW down, all-sky	=	**13**	Surface LW down, clear-sky	=	**12**
TOA LW all-sky	=	**9**	TOA LW clear-sky	=	**10**
G greenhouse effect, all-sky	=	**6**	G greenhouse effect, clear-sky	=	**5**
LWCRE (surface, TOA)	=	**1**	SWCRE (surface)	=	**–2**

Accuracy of the N positions, EBAF Ed4.1, 19 years

Eq. (1) **8** + (**12** – **15**) = **10**/2 Eq. (3) **8** + **12** = 2 × **10**

Eq. (2) **6** + (**13** – **15**) = (**9** – **1**)/2 Eq. (4) **6** + **13** = 2 × **9** + **1**

Clear: SW+LW net = OLR/2	**Clear: SW net + LW down = 2OLR**
211.73 = **8** × 26.68 – 1.71	
317.48 = **12** × 26.68 – 2.68	211.73 = **8** × 26.68 – 1.71
398.46 = **15** × 26.68 – 1.74	317.48 = **12** × 26.68 – 2.68
130.75 = 5 × 26.68 – 2.65	**529.21 = 20 × 26.68 – 4.39**
133.00 = 5 × 26.68 – 0.40	**532.02 = 20 × 26.68 – 1.58**
ΔEq(1) = – 2.25 Wm^{-2}	**ΔEq(3) = – 2.80 Wm^{-2}**
All: SW+LW net = (OLR-LWCRE)/2	**All: SW net + LW down = 2OLR + LWCRE**
163.57 = **6** × 26.68 + 3.47	163.57 = **6** × 26.68 + 3.45
345.15 = **13** × 26.68 – 1.69	345.15 = **13** × 26.68 – 1.69
398.64 = **15** × 26.68 – 1.56	240.22 = **9** × 26.68 + 0.10
240.22 = **9** × 26.68 + 0.10	25.79 = **1** × 26.68 – 0.89
25.79 = **1** × 26.68 – 0.89	**508.72 = 19 × 26.68 + 1.80**
110.05 = 4 × 26.68 + 3.33	**506.23 = 19 × 26.68 – 0.69**
107.21 = 4 × 26.68 + 0.47	**ΔEq(4) = 2.50 Wm^{-2}**
ΔEq(2) = 2.84 Wm^{-2}	

Accuracy of the Greenhouse Effect: Theory and Observation

CERES EBAF Ed4.1, last 12 months

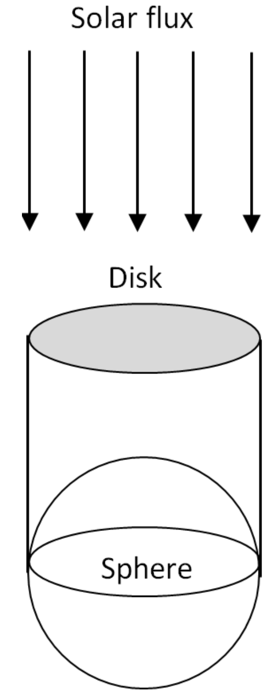
217	406.69	268.74	137.95	0.339202	407.47	243.29	164.18	0.402925
218	408.34	269.87	138.47	0.339105	408.66	244.31	164.35	0.402168
219	407.39	269.3	138.09	0.338963	407.8	243.9	163.9	0.401913
220	403.98	267.77	136.21	0.33717	404.46	242.74	161.72	0.399842
221	399.63	265.56	134.07	0.335485	400.14	240.21	159.93	0.399685
222	393.57	263.56	130.01	0.330335	393.8	237.71	156.09	0.396369
223	391.11	263.08	128.03	0.32735	391.1	237.04	154.06	0.393915
224	390.24	263.34	126.9	0.325185	389.92	237.46	152.46	0.391003
225	392.12	263.67	128.45	0.327578	391.56	238.29	153.27	0.391434
226	396.27	264.54	131.73	0.332425	395.85	238.86	156.99	0.39659
227	399.87	265.53	134.34	0.335959	400.31	239.43	160.88	0.401889
228	403.78	266.9	136.88	0.338996	404.84	241.25	163.59	0.404086
Observed	399.42	265.99	133.43	0.3340	399.66	240.37	159.29	0.3985
1360.68	400.20	266.80	133.40	0.3333	400.20	240.12	160.08	0.4
Theory **51**	**15**	**10**	**5**	1/3	**15**	**9**	**6**	2/5
TSI	ULW_clr	OLR_clr	G_clr	g_clr	ULW_all	OLR_all	G_all	g_all

ULW = **15**, OLR clr = **10** => G (clr) = **5** = 133.40 Wm^{-2} , G (all) = **6** = 160.08 Wm^{-2}

Accuracy of the TOA fluxes

(*clear-sky for total area*, EBAF Ed4.1, 12/2000 – 11/2019)

TSI = 1360.68	**51**	**N** × UNIT	CERES	Diff
LW all-sky	**36** / 4	240.12	240.22	**-0.10**
SW all-sky	**15** / 4	100.05	99.06	**0.99**
LW clear-sky	**40** / 4	266.80	266.01	**0.79**
SW clear-sky	**8** / 4	53.36	53.74	**-0.38**
TOA LW CRE	**4** / 4	26.68	25.79	**0.89**
TOA SW CRE	**-7** / 4	-46.69	-45.30	**-1.39**
TOA Net CRE	**-3** / 4	-20.01	-19.51	**-0.50**



Each flux is an integer on the intercepting cross-section disk

Eq. (5) TSI = **51** = 1360.68 Wm^{-2} => LWCRE = **1** = 26.68 Wm^{-2}

Clear-sky: RSR = **8** ASR = **43** OLR = **40** IMB = **3**

All-sky: RSR = **15** ASR = **36** OLR = **36**

Accuracy of the surface fluxes

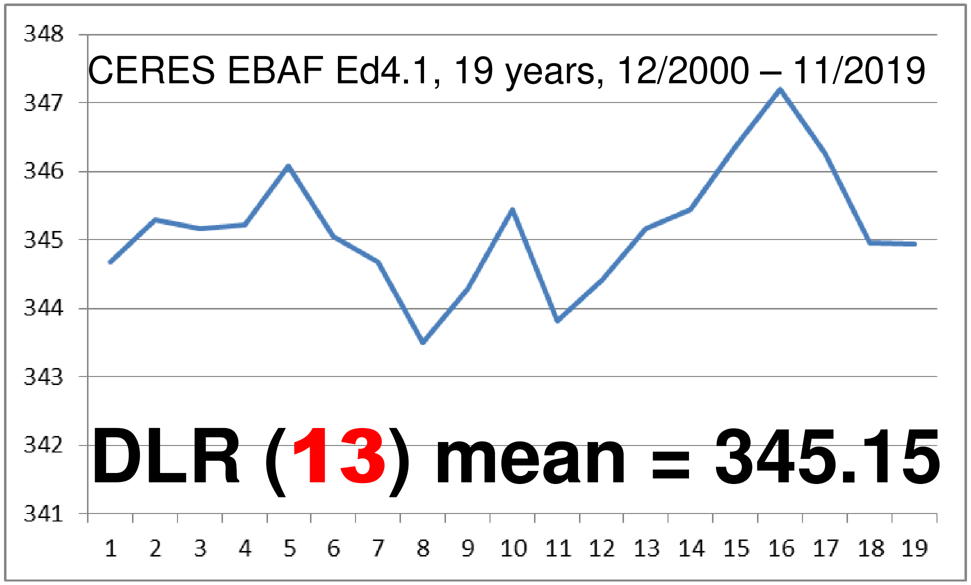
(*clear-sky for total area*, EBAF Ed4.1, 12/2000 – 11/2019)

	N	N × UNIT	CERES	Diff Wm^{-2}
Clear-sky				
LW down	12	320.16	317.48	2.68
LW up	15	400.20	398.46	1.74
SW net	8	213.44	211.73	1.71
All-sky				
LW down	13	346.84	345.15	1.69
LW up	15	400.20	398.67	1.53
SW Net	6	160.08	163.57	-3.49

SFC SW net is not resolved into
downward and upward components

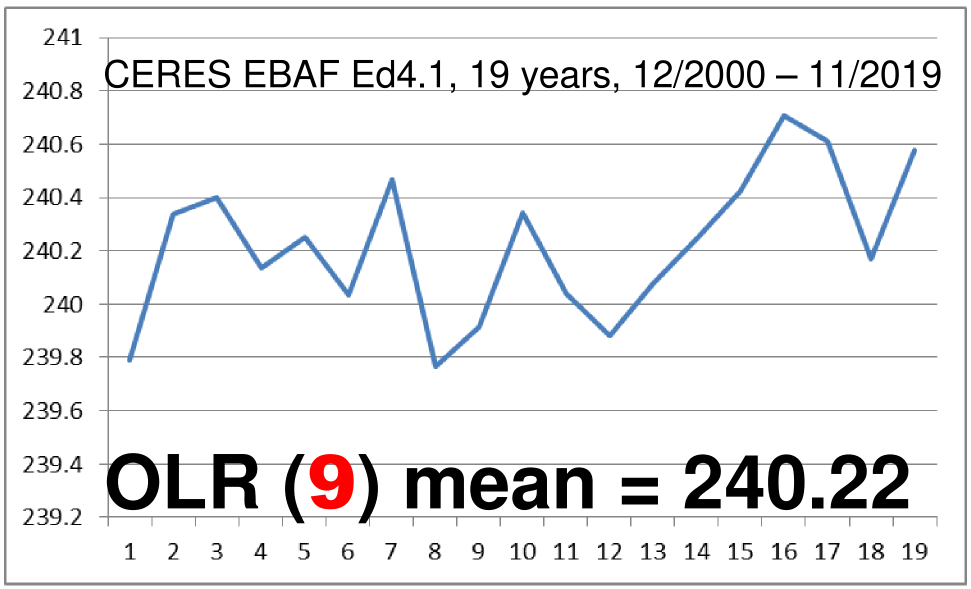
DLR(all-sky) = (13/9)OLR(all-sky) – 1.8 Wm^{-2}

CO_2 increased by 40 ppm during these two decades



Radiative forcing balanced by transfer constraints

ULW = (15/9) OLR according to transfer equations



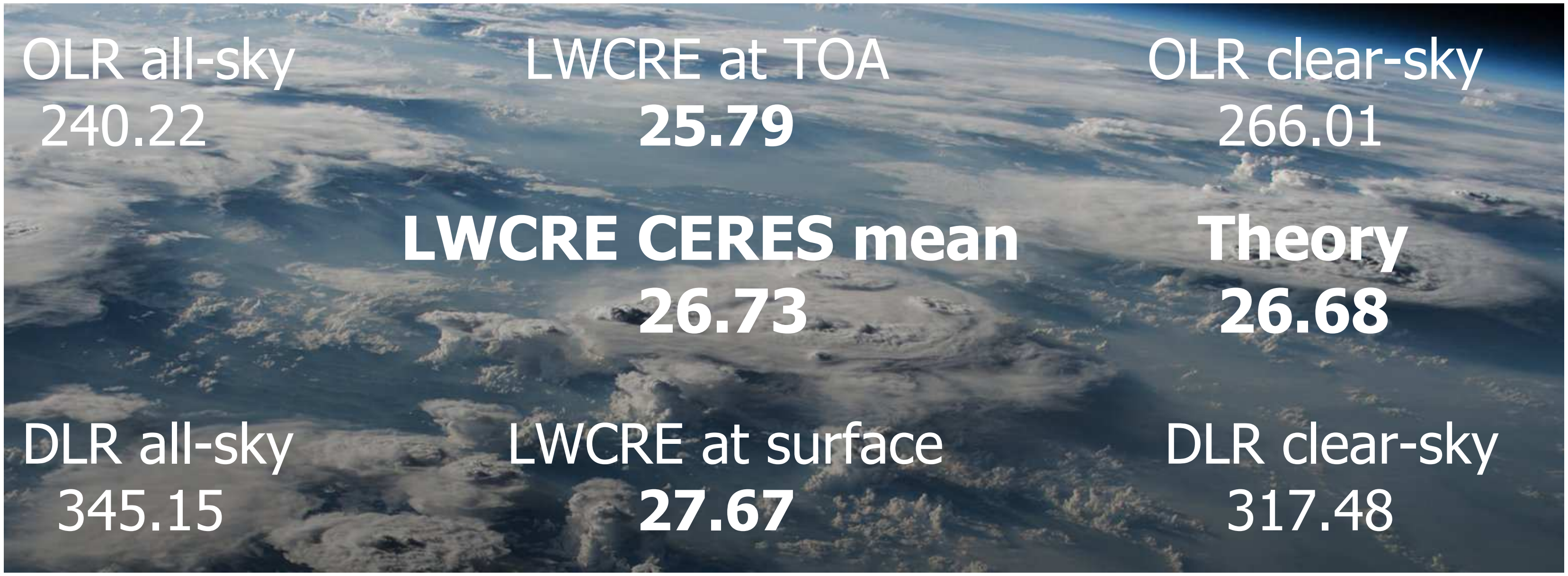
ULW – (15/9) OLR = −1.70 Wm^{-2} according to observation

TSI = 1360.9 Wm^{-2} = 51 => 9 = 240.16 Wm^{-2}

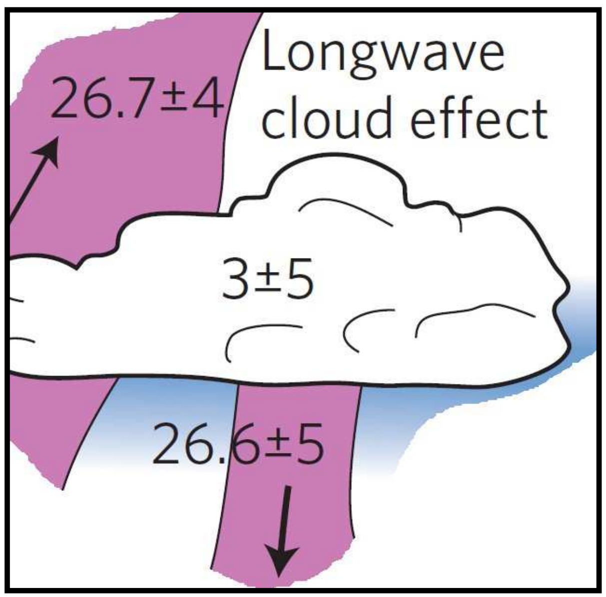
Accuracy of the new clear-sky parameter: no adjustment and with Δ^C adjustment

	TSI 1360.882 = 51 (disk)	N integer	Theory Wm^{-2}	no adj Wm^{-2}	theory – no adj	with Δ^C adjustment	theory – Δ^C adj
ISR	1360.882/4	51/4	340.22	340.0	0.22	340.0	0.22
Clear-Sky	LW	40/4	266.84	268.1	-1.26	266.0	0.84
	SW	8/4	53.37	53.3	0.07	53.8	-0.43
	Net	3/4	20.01	18.6	1.41	20.3	-0.29
CRE	LW	4/4	26.68	27.9	-1.22	25.8	0.78
	SW	-7/4	-46.70	-45.8	-0.90	-45.3	-1.40
	Net	-3/4	-20.01	-17.9	-2.11	-19.6	-0.41
			Surface				
Clear-Sky	LW down	12	320.21	313.9	6.31	317.5	2.71
	LW up	15	400.26	397.6	2.66	398.5	1.76
	LW Net	-3	-80.05	-83.7	3.65	-81.0	0.95
	SW Net	8	213.47	213.5	-0.03	211.7	1.77
	SW+LW Net	5	133.42	129.8	3.62	130.7	2.72

Accuracy of mean CERES LWCRE = 0.05 Wm^{-2}



Stephens et al. (2012)

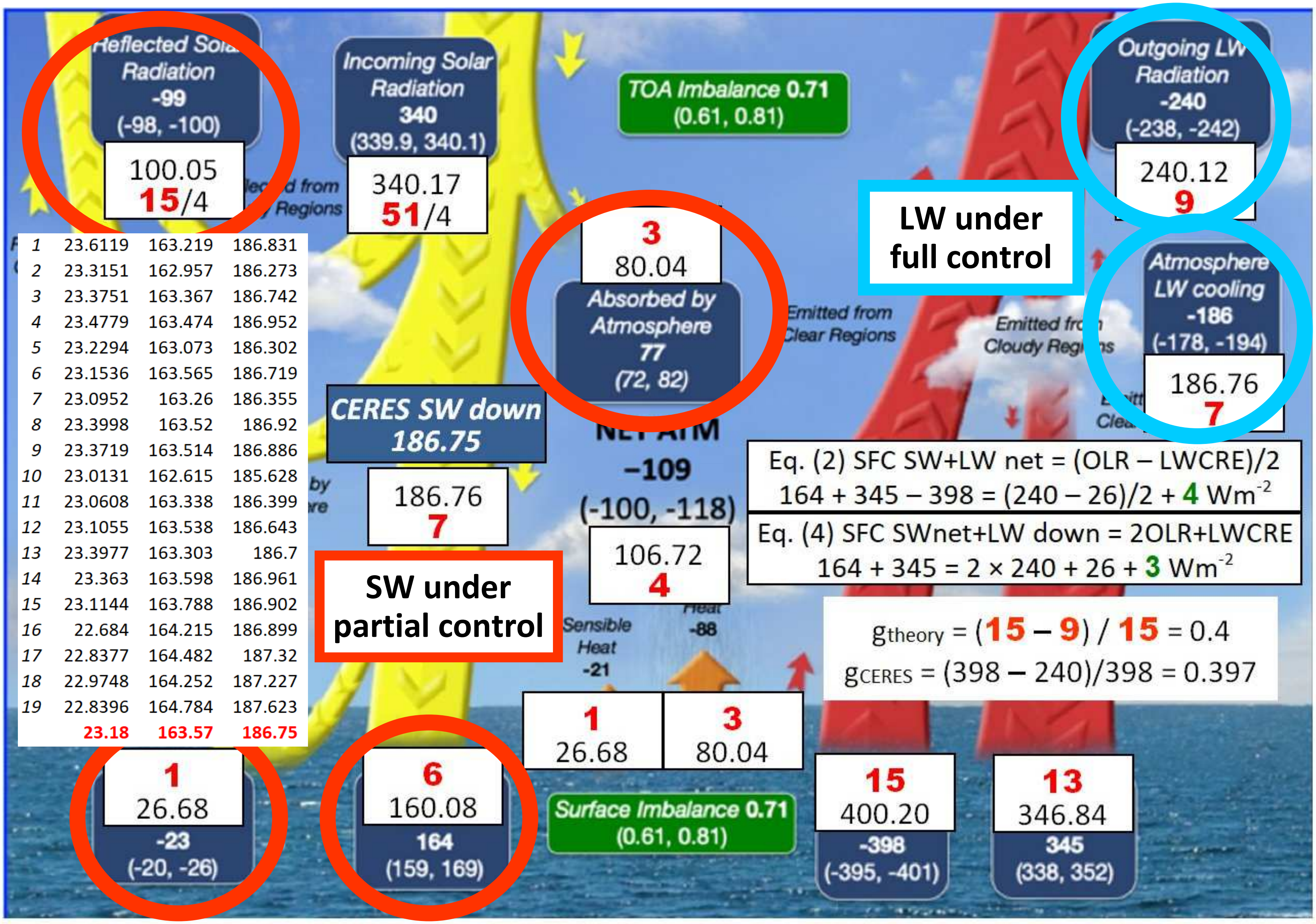


LWCRE Theory

1 = TSI / 51
= 1360.68/51
= 26.68 Wm^{-2}

CERES – Theory:
0.05 Wm^{-2}

The Bluehouse Effect, detected by CERES



1	23.6119	163.219	186.831
2	23.3151	162.957	186.273
3	23.3751	163.367	186.742
4	23.4779	163.474	186.952
5	23.2294	163.073	186.302
6	23.1536	163.565	186.719
7	23.0952	163.26	186.355
8	23.3998	163.52	186.92
9	23.3719	163.514	186.886
10	23.0131	162.615	185.628
11	23.0608	163.338	186.399
12	23.1055	163.538	186.643
13	23.3977	163.303	186.7
14	23.363	163.598	186.961
15	23.1144	163.788	186.902
16	22.684	164.215	186.899
17	22.8377	164.482	187.32
18	22.9748	164.252	187.227
19	22.8396	164.784	187.623
	23.18	163.57	186.75

Eq. (1) – (5): A theoretical steady state for our Aquaplanet

Summary and Conclusions

- Earth's global energy budget is controlled by radiation transfer equations originated in Schwarzschild's theory. Eq. (1) and (2) may be derived from first principles.
- Each of the four equations is satisfied by two decades of CERES observations within $\pm$ 3 Wm^{-2}. Forcing and feedbacks are expected to act within these limits.
- The fundamental individual fluxes (both SW and LW) are within ± 1 Wm^{-2}.
- The accuracy of CERES data (fit to theory) is much better than indicated in DQS.
- There are other constraints: the extension of the **N** system to total solar irradiance is unexpected, but extremely precise:
- Eq. (5) LWCRE = **1** = TSI / 51 $\pm$ 0.01 Wm^{-2}.
  LWCRE = 26.68 Wm^{-2} (SORCE TSI) or 26.69 Wm^{-2} (TSIS1) .
- Eq. (6) 2ASR = 2OLR + WIN – LWCRE is a valid equation as well
  (not detailed here, see EGU2020 display).
- I expect $\pm$ 3 Wm^{-2} fluctuations around, but not systematic deviation from, the equilibrium positions in the forthcoming decades.
- Open questions: limits, tipping points, shifts, ice ages (albedo?)

Thank you CERES Science Team for the excellent work!